Photodynamic Notes, VIII. By Pliny Earle Chase, LL.D.

(Read before the American Philosophical Society, May 18, 1883.)

376. *Virials.*

The theory of the virial, or mean *vis viva* during stationary motion, enables us to coördinate all forms of cyclical motion : rotary, orbital and oscillatory. The grandest manifestations of the virial, which are given in cosmical motion, must be governed by the same laws as govern molecular movements. The complete development of the theory should, therefore, remove all the obscurity which still clings to the doctrine of radiodynamic unity. The science of comparative kinetics is greatly indebted to Clausius, for his presentation of the theory, for the consequent simplicity which it introduces into the solution of problems which would otherwise be exceedingly complicated and for the facility of explanation, which it gives for methods which are substantially the same, but which, on account of their novelty, have been often misunderstood.

377. *Virial Postulates.*

My photodynamic and other physical researches have been rewarded by a great number of cosmical illustrations of virial efficiency, which are based upon the following postulates :

1. That cosmical masses represent internal energies, such as would be found if they were condensed from some primitive tenuous, elastic form of matter.

2. That all chemical elements may have been condensed, in like manner, from a single primitive element or æther.

3. That the velocity which enters into the primitive radial virial of the oscillating æthereal particles is the velocity of light (v_{λ}).

4. That the stationary motions of central inert masses, which represent the equal actions and reactions of primitive and derived virials should continue until the velocity of the primitive virial has been alternately acquired and lost.

5. That all stationary motions which represent equal actions and reactions should be harmonic.

378. *Stellar Virials.*

Solar or Stellar centres of planetary systems are central inert masses (Post. 4), which are endowed with velocities of stationary motion, tending to give velocities of stationary revolution, sending forth æthereal oscillations with the velocity of light (Post. 3) and representing internal energies like those which would spring from nebular condensation (Post. 1). Their central stationary motions should, therefore, be cyclically determined by the alternate acquisition and exhaustion of the radial velocity of light (Post. 4). Herschel (*Outlines of Astronomy,* Sect. 399) discoursed eloquently on the Sun's rays as "the ultimate source of almost every motion which takes place on the surface of the earth." We may, therefore, reasonably look to them for evidences of virial efficiency, in various forms, which will furnish satisfactory proof of radiodynamic unity.

379. *Equal Virial Action and Reaction.*

Circular orbital velocity which is due to solar action may be represented by the equation

$$v_n = \sqrt{g_n r_n} \qquad 1.$$

The limiting value of v_n, which it cannot exceed, is found at Sun's surface (r_o), where g_o is a maximum. It may be represented by

$$[v_o] = \sqrt{g_o r_o} \qquad 2.$$

The third and fourth postulates lead to the equation

$$v_\lambda = g_o t_o \qquad 3.$$

This equation should hold good for all values of r in an expanding or contracting nucleus, inasmuch as g varies inversely as r^2 and the principle of conservation of areas requires that the time of rotation should vary directly as r^2. The product of the two factors should, therefore, be constant.

380. *Numerical Verification.*

Taking Sun's semi-diameter (r_o) as the unit of length, and the British Nautical Almanac estimate of Sun's apparent semi-diameter (961.″83) as the parallactic unit, we find, for Earth's semi-axis major

$$\rho_3 = 214.45 r_o \qquad 4.$$

Earth's mean orbital velocity (1) may be found by dividing $2\pi\rho_3$ by the number of seconds in a year (31558149). This gives

$$v_3 = .0000001990099 \rho_3 \qquad 5.$$

This value varies slightly with varying orbital eccentricity. The greatest secular range of variation, however, is less than $\frac{1}{8}$ of one per cent.

Circular orbital velocity varying inversely as the square root of the radius-vector, we find (2), (4), (5)

$$[v_o] = .0000291562 \rho_3 = .006252 55 r_o \qquad 6.$$

$$g_o = .0000003909445 r_o \qquad 7.$$

Struve's constant of aberration gives, by (3) and (7)

$$v_\lambda = g_o t_o = 214.45 r_o \div 497.827 = .430772 r_o \qquad 8.$$

$$t_o = 1101876 \text{ sec.} = 12.753 \text{ days} \qquad 9.$$

This gives for a double oscillation, or complete rotation of Sun, 25.506 days. Laplace's estimate was 25.5 days. The motion of sun-spots near the equator is accelerated by centrifugal force, tendencies to orbital velocity, "repulsion," or some other unknown influence. Spörer's formula gives 24.62 days for the period at the equator, where no spots have ever been observed. His third estimate, for 1866, was 25.234 days.

381. *Virials of Rotation.*

The rotating æthereal tendency of stationary motion, which is limited by equations (2) and (3), gives the following value for the limiting radius (ρ_λ) of orbital and æthereal tendencies :

$$\rho_\lambda = \frac{v_\lambda}{[v_0]} r_0 = v_\lambda \sqrt{\frac{r_0}{g_0}} \qquad 10.$$

Laplace's limit (l) of equal rotary and orbital velocity is given by the equation

$$l = \left(\frac{\rho_\lambda}{\pi r_0}\right)^{\frac{3}{2}} r_0 = \left(\frac{\rho_\lambda}{\pi}\right)^{\frac{3}{2}} r_0^{-\frac{1}{2}} \qquad 11.$$

The limit at which the equatorial velocity of stationary motion would give v_0 is

$$\left(\frac{l}{r_0}\right)^{\frac{2}{3}} r_0 = l^{\frac{2}{3}} \div r_0^{\frac{1}{3}} = \rho_\lambda \div \pi \qquad 12.$$

The limit at which the equatorial velocity of stationary motion would give v_λ, as deduced from (10) and (12), is

$$[l] = \rho_\lambda^2 \div \pi r_0 \qquad 13.$$

The limit of a homogeneous, elastic, æthereal atmosphere which would propagate undulations with the velocity of light, is

$$M = \pi [l] = \rho_\lambda^2 \div r_0 \qquad 14.$$

382. *Virial Centres of Oscillation.*

The virials of rotating tendency must influence grosser inert particles or masses, as well as the æthereal atmosphere. Loci of important oscillatory influence may be found at radii of mean æthereal momentum (ρ_α), of linear oscillation (ρ_β), of reciprocal linear oscillation (ρ_γ), of spherical oscillation (ρ_δ), and of reciprocal spherical oscillation (ρ_ϵ). Taking ρ_λ as the common virial locus of these several oscillating tendencies, we have

$$\rho_\alpha = 2\rho_\lambda \qquad 15.$$

$$\rho_\beta = 1.5\rho_\lambda \qquad 16.$$

$$\rho_\gamma = 3\rho_\lambda \qquad 17.$$

$$\rho_\delta = 2.5\rho_\lambda \qquad 18.$$

$$\rho_\varepsilon = \tfrac{5}{3}\rho_\lambda \qquad 19.$$

All of these forms of action and reaction must be called into play by solar and stellar radiation, and they should all be studied in investigating the maintenance of cosmical energy.

383. *Maintained Vibrations.*

Lord Rayleigh (*Phil Mag.*, April, 1883) discusses a vibrating system which is subject to dissipative forces, and the necessity, when the vibrations are maintained, that the vibrating body should be in connection with a source of energy. In the usual equation

$$\frac{d^2\theta}{dt^2} + \frac{d\theta}{dt} + n^2\theta = 0 \qquad 20.$$

two principal classes of maintained vibrations may be distinguished; the more extensive class being that in which the magnitude and phase of the sustaining force depend in an approximately constant manner, upon the amplitude and phase of the vibration itself. The only case in which, according to (20), a steady vibration is possible, is when the complete value of κ is zero. If any portion of the energy of cosmical masses is dissipated, æthereal energy must be proportionately increased. The æther accordingly becomes a "source of energy," and although we are not yet able to see fully how the connection of this source with solar radiations is kept up, the equivalence of v_λ to v_γ (Note 321) shows that it is kept up, through cyclical actions which cover a period of about $12\frac{3}{4}$ days.

384. *Virials of Wave Propagation.*

It has often been tacitly assumed that there is no actual radial oscillation in luminous radiation, like that of the atmospheric particles in the propagation of sound-waves. In 1872 (*Proc. Amer. Phil. Soc.*, xii, 394) I showed that the secondary centre of oscillation, on returning from the centre of linear oscillation towards the linear centre, is at $\frac{5}{9}$ of the extreme excursion. Hence the tangential virial of an oscillating æthereal particle (μ_a), is $\frac{9}{5}$ of the radial virial of the same particle (μ_β). More than five years afterwards (*Phil. Mag.* [5], iii, 453; iv, 209), Maxwell stated that the ratio of the virial velocity is $\sqrt{\frac{9}{5}}$, but he gave no reason for his inference and none has yet been found among his unpublished papers. His statement and mine are substantially identical, the only difference being that he looked to the relative mean momentum of the oscillating particles, while I looked to their relative virials.

$$\mu_a = 1.8\ \mu_\beta \qquad 21.$$

385. *Time-Relations of Inertia.*

The question of instantaneous action is still, and probably will long continue to be, a mooted one. The most impressive form in which it has ever been presented, is Laplace's statement that gravitating action requires a velocity which is more than 100,000,000 times as great as the velocity of light, and that it may be assumed to be absolutely instantaneous at all distances. It is sometimes said that inertia is instantaneously overcome. This may, perhaps, be true in some sense, but we cannot know that it is so, until we know more than we have yet learned about the way in which velocity is transferred from one body to another. In general physical investigations the element of time, usually in the form of time-integrals, requires consideration whenever there is any change of motion.

386. *Correlation of Virials.*

Questions of kinetic unity and correlation are greatly complicated by differences of inertia and by the lack of generally recognized standards of comparison. If all forms of force are transmitted through æthereal intervention, all virials should be capable of representation in terms of æthereal mass and velocity. The velocity of luminous undulation then becomes a natural standard of velocity. Whenever velocity is imparted or destroyed by gradual accelerations or retardations (f), a time can always be found which will give the equation

$$ft = v_\lambda \qquad 22.$$

By cöordinating the times which are required by this equation in different forms of energy, the evidences of primitive kinetic unity may be multiplied indefinitely.

387. *A Natural Unit of Time.*

Errors of measurement which are of any specific magnitude, increase in relative importance inversely as the magnitude which is measured. An error of .0001 inch in any of the dimensions of a microscopic object would be very serious, but in an object which is a foot or more in length it would be insignificant. It is desirable, therefore, in studying kinetic unity, to begin with phenomena which involve kinetic maxima. The most far-reaching acceleration of which we can make measurements, is that of gravitation, and the greatest gravitating acceleration of which we have any direct knowledge (g_0) is found at Sun's surface. Substituting in (22) we have

$$ft = g_0 t_0 = v_\lambda \qquad 23.$$

Therefore, Laplace's principle of periodicity (Note 333), the collateral hypotheses of various investigators (Note 278), the fourth virial postulate (Note 377), the considerations which make v_λ a natural unit of velocity (Note 386), as well as many other correlations of photodynamic and general cyclical energy, point to the time of solar rotary oscillation as a natural unit of time.

388. *Virial Transfers.*

An energy which is wholly transferred from one æthereal mass to another equivalent æthereal mass, must be accompanied by a like transfer of velocity, whether the transfer is in the form of potential (v_α), work (v_β), gravitation (v_γ), torsion (v_δ), electricity (v_ϵ), rotation (v_ζ), revolution (v_η), heat (v_θ), chemical affinity (v_κ), or luminous undulation (v_λ). We have, therefore, for limiting velocities when all the units are homologous,

$$v_\alpha = v_\beta = v_\gamma = v_\delta = v_\epsilon = v_\zeta = v_\eta = v_\theta = v_\kappa = v_\lambda \qquad 24.$$

In cyclical movements which are due to virial tranfers, these several equivalents may be indicated by equations which are based on the third and fourth postulates (Note 377) and which are analogous to (3).

389. *Cardinal Limits.*

In seeking further numerical verifications of the foregoing virial equations, we find the photodynamic limiting radius of orbital and æthereal tendencies (10) by substituting (6) and (8).

$$\rho_\lambda = 688.954\, r_0 = 3.212654 \rho_3 \qquad 25.$$

Substituting (25) in (11), we get for Laplace's limit

$$l = 36.366 r_0 \qquad 26.$$

The substitution of (25) in (12) gives

$$\rho_\lambda \div \pi = 219.301 r_0 = 1.0226 \rho_3 \qquad 27.$$

Hence by (13), we find for the locus of v_λ in solar rotation,

$$[l] = 151088.1 r_0 = 704.538 \rho_3 \qquad 28.$$

And the solar modulus of light (14) is

$$M = 474657.3\, r_0 = 2213.37\, \rho_3 \qquad 29.$$

390. *Influence of Synchronous Radial and Tangential Virials.*

The theoretical variation of æthereal density within the limits of our planetary system (Note 240) is so slight that the several vector radii may be considered as indicative of virial projection against a resistance which is nearly uniform. The radial and tangential virials (Note 384) being synchronous, we may with reason look for cosmical evidences of the synchronism. Accordingly we find, from (21) and (25), the following regular series of approximations to planetary loci. The subscripts, 1, 2, 3, denote, respectively, secular perihelion, mean, secular aphelion.

$1{\cdot}8^{-4}\,\rho_\lambda$	=	$\cdot3060\rho_3$	Mercury_1	=	$\cdot2974\rho_3$	30.
$1{\cdot}8^{-3}\,\rho_\lambda$	=	$\cdot5509\rho_3$	Venus_1	=	$\cdot6722\rho_3$	31.
$1{\cdot}8^{-2}\,\rho_\lambda$	=	$\cdot9916\rho_3$	Earth_2	=	$1{\cdot}0000\rho_3$	32.
$1{\cdot}8^{-1}\,\rho_\lambda$	=	$1{\cdot}7848\rho_3$	Mars_3	=	$1{\cdot}7365\rho_3$	33.
$1{\cdot}8^{0}\,\rho_\lambda$	=	$3{\cdot}2127\rho_3$	Asteroid 108	=	$3{\cdot}2120\rho_3$	34.
$1{\cdot}8^{1}\,\rho_\lambda$	=	$5{\cdot}7828\rho_3$	Jupiter_3	=	$5{\cdot}5193\rho_3$	35.
$1{\cdot}8^{2}\,\rho_\lambda$	=	$10{\cdot}4090\rho_3$	Saturn_2	=	$10{\cdot}3433\rho_3$	36.
$1{\cdot}8^{3}\,\rho_\lambda$	=	$18{\cdot}7362\rho_3$	Uranus_2	=	$19{\cdot}1836\rho_3$	37.
$1{\cdot}8^{4}\,\rho_\lambda$	=	$33{\cdot}7252\rho_3$	Neptune_3	=	$30{\cdot}4696\rho_3$	38.
Geom'l Mean	=	$3{\cdot}2127\rho_3$	Geom'l Mean	=	$3{\cdot}2200\rho_3$	39.

All of these approximations represent loci of belt-condensation, for the respective planets, which are in accordance with the nebular hypothesis. The geometrical means differ by less than $\frac{1}{4}$ of one per cent. The photodynamic mean represents the semi-axis major of Asteroid 108; the planetary mean, the semi-axis major of Asteroid 122. The second photodynamic locus ($\cdot5509\rho_3$) is, within less than one per cent., the arithmetical mean between the semi-axes major of Mercury and Venus ($\cdot5552$).

391. *Photodynamic Centre of Various Oscillations.*

The common virial locus (Note 382) of mean momentum, linear oscillation, spherical oscillation, and reciprocal oscillations, gives the following planetary approximations by (15), (16), (17), (18), (19) and (25):

ρ_α	=	$6{\cdot}4253\rho_3$	Cardinal centre	=	$6.4451\rho_3$	40.
ρ_β	=	$4{\cdot}8190\rho_3$	Jupiter_1	=	$4{\cdot}8863\rho_3$	41.
ρ_γ	=	$9{\cdot}6380\rho_3$	Saturn_2	=	$9{\cdot}5389\rho_3$	42.
ρ_δ	=	$8{\cdot}0318\rho_3$	.4 Uranus_3	=	$8{\cdot}2717\rho_3$	43.
ρ_ε	=	$5{\cdot}3545\rho_3$	Jupiter_2	=	$5{\cdot}2028\rho_3$	44.
Ar. Mean	=	$6{\cdot}8537\rho_3$	Ar. Mean	=	$6{\cdot}8690\rho_3$	45.
Ge. Mean	=	$6{\cdot}6325\rho_3$	Ge. Mean	=	$6{\cdot}6421\rho_3$	46.

It will be seen from (43) that the second locus of spherical rotary projection from ρ_λ, $(2{\cdot}5 \times 2{\cdot}5\rho_\lambda = 20{\cdot}0795\rho_3)$, is within the secular orbital range of Uranus. The cardinal centre (40) is the centre of gravity, at conjunction, of Saturn_2 and Jupiter_3. It represents, therefore, the locus of mean rotary momentum for their combined masses, at the time of Jupiter's incipient rupturing subsidence, according to Herschel's modification of the nebular hypothesis.

392. *Further Relations of the Cardinal Centre.*

The cardinal centre, which introduces the series in the foregoing note, also represents important relations to the following additional virial loci :

$$\rho_\zeta = \sqrt{\Sigma m\rho^2 \div \Sigma m} = 9.2443\rho_3 \qquad 47.$$

$$\rho_\eta = \Sigma m\rho \div \Sigma m = 7{\cdot}5228\rho_3 \qquad 48.$$

$$\rho_\theta = \tfrac{1}{2}\,(\text{Saturn}_2 + \text{Jupiter}_3) = 7{\cdot}5291\rho_3 \qquad 49.$$

$$\rho_\iota = \tfrac{1}{2}\,(6{\cdot}4451 + 8{\cdot}2717) = 7{\cdot}3584\rho_3 \qquad 50.$$

$$\rho_\kappa = \tfrac{1}{4}\,\text{Neptune}_2 = 7{\cdot}5084\rho_3 \qquad 51.$$

The locus of mean planetary nebular inertia (47) is in Saturn's orbit, where the rings, the satellite system and the specific gravity bear witness to the results of nebular condensation. The locus of mean planetary nebular momentum (48) approximates closely to the arithmetical mean between Saturn_2 and Jupiter_3 (49), to the arithmetical mean between the cardinal centre and the incipient virial locus of spherical rotation for Uranus (50), and to the virial locus for the mean linear momentum of Neptune's semi-axis major (51).

393. *Primitive Virial Influence on Mass.*

The virial radius of mean momentum not only determines the centre of gravity of the two chief planetary masses (15), (40), but it also determines the relative masses of Sun (m_0) and Jupiter (m_5) at initial nebular rupture (secular perihelion). We find, accordingly,

$$m_0 r_0 = m_5 \rho_{5\cdot 1} \qquad 52.$$

Stockwell's estimate of Jupiter's secular eccentricity is ·0608274. This gives $\rho_{5\cdot1} = {\cdot}9391726 \times 5{\cdot}202798 \times 214{\cdot}45 = 1047{\cdot}872 r_0$. Therefore (52) :

$$m_0 = 1047.872\, m_5 \qquad 53.$$

Bessel's estimate is 1047·879. This harmony is the more significant because Jupiter's nebular locus of incipient rupture (4·8863) is central between the loci of incipient subsidence of Uranus (20·6792) and Neptune (30·4696) at opposition.

$$\rho_{5\cdot1} = \tfrac{1}{2}\,(\rho_{7\cdot3} - \rho_{6\cdot3}) \qquad 54.$$

394. *Successive Orders of Photodynamic Influence.*

While Jupiter traverses the primitive nebular centre, Earth traverses the centre of the belt of greatest condensation.

$$\tfrac{1}{2}\,(\rho_{1\cdot1} + \rho_{4\cdot3}) = \rho_3 \qquad 55.$$

Stockwell's estimates for the secular limits of the dense belt (Mercury$_1$ and Mars$_3$) are, $\rho_{1\cdot 1} = \cdot 2974$; $\rho_{4\cdot 3} = 1\cdot 7365$. This gives for (55) $1\cdot 0169\ \rho_3$, which is nearly $\frac{1}{\pi}$ of the mean proportional (27) between Sun's radius (r_0) and the solar modulus of light (29). These successive indications of virial influence upon Saturn and Jupiter (40), Sun and Jupiter (53), Uranus and Neptune (54), and the relative positions of the dense planets, are full of suggestive interest.

395. *Virials of Secondary Rotations.*

While the rotation of the chief nucleal centre (Sun) is determined by the velocity of light (3), the rotations of the secondary centres of nebulosity (Jupiter) and condensation (Earth) are determined, respectively, by circular orbital velocities at Sun's surface $[v_0]$ and at the mean centre of gravity of Sun and Jupiter $[v_a]$.

$$g_5 t_5 = [v_0] = \sqrt{g_0 r_0} \qquad 56.$$

$$g_3 t_3 = [v_a] = \sqrt{g_a r_a} \qquad 57.$$

The data for the solution of (57) have been more accurately and satisfactorily determined than for (56).

$$g_3 t_3 = \frac{32\cdot 088}{5280} \times \frac{86164\cdot 08}{2} = 261\cdot 821 \text{ miles} \qquad 58.$$

396. *Jupiter's Diameter and Density.*

Circular orbital velocity varying inversely as $\sqrt{r}$, we find (52), (53), (57), (58)

$$g_5 t_5 = [v_0] = g_3 t_3 \div \sqrt{.9391726} = 270\cdot 167 \text{ miles} \qquad 59.$$

$$[v_3] = [v_0] \div \sqrt{214\cdot 45} = 18\cdot 449 \text{ miles} \qquad 60.$$

Hall's estimate of the period of Jupiter's rotation ($9^h\ 55^m\ 26^s.5$) gives

$$t_5 = 4^h\ 57^m\ 43.25 = 17863.25 \text{ seconds} \qquad 61.$$

Substituting this value in (59) we find

$$g_5 = 79\cdot 856 \text{ ft.} = 2\cdot 4887\ g_3 \qquad 62.$$

Hence, by (53) and (69)

$$m_5 = 315\cdot 38\ m_3 \qquad 63.$$

$$r_5 = 11\cdot 257 r_3 \qquad 64.$$

$$\delta_5 = \cdot 2211 \delta_3 \qquad 65.$$

Different estimates of Jupiter's mean apparent semi-diameter give values for r_5 ranging between $10\cdot 8 r_3$ and $11\cdot 5 r_3$

397. *Sun's Mass and Distance.*

Earth's gravitating acceleration and its orbital velocity (60) being known, we have all the data which are needed for estimating Sun's relative mass and mean distance.

$$\rho_3 = 31{,}558{,}149\ [v_3] \div 2\pi = 92{,}662{,}000 \text{ miles} \qquad 66.$$

$$r_0 = \rho_3 \div 214\cdot 45 = 432{,}090 \text{ miles} \qquad 67.$$

At Earth's surface, $\sqrt{gr} = 4{\cdot}9073$. It varies as $\sqrt{\frac{m}{r}}$. Therefore (60)

$$\frac{m_0}{\rho_3} : \frac{m_3}{r_3} : : 18{\cdot}449^2 : 4{\cdot}9073^2 \qquad 68.$$

$$m_0 : m_3 : : 330482 : 1 \qquad 69.$$

All of the results which have been drawn from (3), (56), and (57) involve the principle of persistency of vibrations, by which waves tend to propagate themselves indefinitely, with the velocity which is due to their locus of origination.

398. *Masses of Earth and Venus.*

The influence of Jupiter's locus of incipient subsidence on the comparative masses of Jupiter and Saturn, finds some analogy in the two chief planets of the dense belt, Earth and Venus.

$$m_2\rho_{2\cdot3} = m_3\rho_3 \qquad 70.$$

Substituting Stockwell's estimate of the secular aphelion of Venus ($\rho_{2\cdot3} = {\cdot}7744234\rho_3$) in (69), (70).

$$m_0 = 426750m_2 \qquad 71.$$

Hill's estimate is 427240, which differs from (71) by less than $\frac{1}{8}$ of one per cent. The combined virial estimate of Earth's relative mass (69) differs from the purely oscillatory estimate (Note 23) by less than $\frac{2}{5}$ of one per cent.

399. *Comparisons of Potential.*

In order to test the numerical accuracy of the general equation of kinetic-velocities (24) we may begin with the consideration of potential energy, which has been largely treated in thermodynamics. Gravitating potential is usually measured by the height of possible fall, or of *virtual* fall, since the heights which are considered are commonly so small that the variation of g is insignificant. The time of fall (t_a), or the velocity which would be communicated by the fall (v_a), might be taken with equal propriety as the basis of measurement and comparison. The cosmical determination of Joule's equivalent (*Proc. Am. Phil. Soc.*, xix, 20), shows the importance and advantage of adopting fundamental units which can be readily employed in the greatest possible variety of directions.

The general equation of fundamental velocity (24) rests on Laplace's principle of periodicity, "that the state of a system of bodies becomes periodic when the effort of primitive conditions of movement has disappeared by the action of resistances." Hence (3), (8).

$$v_\lambda = v_\gamma \qquad 72.$$

Moreover, the natural standards of time, gravitating acceleration, distance, oscillation and undulatory velocity which are indicated by the solar periodicity of synchronous rotation and evolution at Laplace's limit, solar superficial attraction, Sun's semi-diameter, and luminous radiation, obviously give the following further equality :

$$v_\alpha = v_\beta = v_\zeta = v_\eta = v_\theta = v_\lambda \qquad 73.$$

400. *Completion of Correlation.*

In Coulomb's formula of torsional elasticity (Note 162), if we substitute $\frac{m}{2}$ for f, $gt^2 = M$ (29) and

$$gt = v_\lambda = v_\delta \qquad 74.$$

The investigations of Weber, Kohlrausch, Thomson, Maxwell, Ayrton, and Perry have shown that

$$v_\lambda = v_\varepsilon \qquad 75.$$

Notes 16, 90–3, 97 give various ways of coördinating chemical and cosmical actions with luminous undulation, so as to get the equation

$$v_\kappa = v_\lambda \qquad 76.$$

In throwing a ball into the air, the thermal equivalent of projectile force is equivalent to the product of the mass by the sum of the retarding resistances. In solar superficial radiation, the gravitating reaction is exhausted in a half rotation. By a simple extension of these principles we have deduced equations 73–6, which, when combined, give a complete practical verification of the general kinetic correlation (24).

401. *Phyllotaxy and Harmony of Absorption Bands.*

Langley's observations with the spectro-bolometer, at Allegheny Observatory and on the summit of Mt. Whitney, show four remarkable absorption bands in the infra-red portion of the solar spectrum, at 0.μ94, 1.μ14, 1.μ37 and 1.μ83. These wave-lengths are very nearly proportional to the numbers 4, 5, 6, 8, as is shown by the following table:

	Harmonic.	Observed.
α	·92	·94
β	1·15	1·14
γ	1·38	1·37
δ	1·84	1·83

They give, therefore, the following phyllotactic approximations: $\alpha \div \beta = 2 \times \frac{2}{5}$; $\alpha \div \gamma = \frac{2}{3}$; $\alpha \div \delta = \frac{1}{2}$; $\beta \div \delta = \frac{5}{8}$. The phyllotactic harmony is modified by an approximate repetition of the harmonic ratio $\frac{5}{6}$.

	Harmonic.	Observed.
α_0	95	.94
β_0	114	1.14
γ_0	136.8	1.37

We thus find, wherever we look, abundant evidence, not only of primary harmonic influence, but also of secondary and subordinate modifications which need to be carefully studied in connection with virial researches.

402. *Consequences of Ferrel's Law.*

The science of Meteorology may, for many good reasons, be regarded as a peculiarly American science. William Ferrel's discussion of the motion of fluids and solids relative to the Earth's surface, which was first published in the summer of 1856, placed the laws of cyclonism and anticylonism on a solid mathematical basis. He showed that, in the northern hemisphere, all moving bodies are constantly subjected, in consequence of the Earth's rotation, to a deflection towards the right hand. Hence all atmospheric surface currents which are mainly governed by a downward pressure, tend to curve in the direction of the hands of a watch, or successively through north, east, south, west. All surface currents which are mainly governed by an upward pressure, tend to flow in an opposite direction, or through north, west, south, east.* The heavy winds are called anticyclonic; the light winds, cyclonic.

There can be no descending currents in one place without ascending currents in another; therefore, in every atmospheric disturbance, there must be simultaneous cyclonic and anticyclonic winds. Such disturbances originate either in an unusual cooling and condensation, or in an unusual heating and expansion of air. In the former case the inflow, in the upper regions of the atmosphere, will produce an increased pressure. In the latter, the outflow will produce a diminution of pressure. In the restoration of equilibrium, currents of warm air are often brought into contact with colder currents. If the currents are both saturated with moisture, or if they contain more vapor than can be retained under the temperature of the mixed currents, precipitation takes place, in the form of rain, hail, or snow. This precipitation reduces the weight of the atmospheric column and the barometer falls. Accordingly, there is a constantly increasing tendency to cyclonism about storm centres, and there has been a very prevalent disposition to look upon all storms as of cyclonic origin.

A little reflection, however, will show that the initial mixture of currents may be due to either of the causes above mentioned; either to the flow of warmer air into a cold depression at the top of the atmosphere, or to a flow of cold air, at the earth's surface, towards a region of low barometric pressure. In the former case, the initial superficial currents are determined by a downward pressure, and they are, therefore, anticyclonic; in the latter they are determined by an upward pressure and are cyclonic.

A careful study of the weather maps shows that the heaviest rains and snows occur in advance of the centres of low barometric pressure, or in the rear of the centres of high barometric pressure. If storms began in the cyclonic currents, the reverse should be true; the greatest effect following the low centre and preceding the high centre.

The frequent failures of forecasts, during the past winter, seem to have been mainly due to a misinterpretation or a misconception of these facts, to which the writer first called attention in 1871.

403. *Study of Stormy Anticyclonism.*

Loomis's discussions (Note 367) show the need of watching the development of storms at all stages, from the first indications of atmospheric

*This will be evident, if we imagine ourselves to be lying in the current and facing the direction towards which the pressure tends.

disturbance, until the restoration of fair weather. The limit between anticyclonic and cyclonic tendencies, may be approximately assumed to be midway between the centres of high and low barometric pressure. All cloudiness or precipitation between the limit and the high centre, represents anticyclonic influence ; all between the limit and the low centre represents cyclonic influence. Local cyclonism sets in soon after precipitation begins, and the anticyclonic influence is thus partially hidden ; but a critical examination of the weather maps will show that the prevailing currents of the region often continue to be anticyclonic until the rain or snow is nearly, or quite over. The evidences of storm breeding and stormy anticyclonism will be still more striking, if the changes of barometric pressure are studied in connection with the beginnings and subsequent growth of cirrus, cumulus, and nimbus clouds, as well as with the rainfall and the final breaking up of cloudiness. There are good reasons for believing that such study, systematically and thoroughly continued under the direction and with the facilities of the Signal Service Bureau, would raise the successful verifications of the Washington forecasts to an average of at least ninety-five per cent.

404. *Pressure of Warm Air.*

Dr. Köppen, in discussing Ley's work on the winds prevailing in Western Europe, announces four new theorems (*Ann. hydr. und magnet. marit. meteor.*, 1882 ; cited by *Science*, **499**). 1. The air-currents deviate from the isobars towards the side of the lower pressure in the lower atmosphere, and of the higher pressure in the upper atmosphere. 2. An excess of pressure exists upon the side of the warmer air-columns. 3. The depressions advance approximately in the direction of the air-current which has a preponderance of accumulated energy. 4. The state of motion of a certain mean layer, of which the height is still to be determined, can in general be substituted for the onward movement of the vortex. A systematic comparison of these propositions with observations and with Blasius's discussion of äerial currents (*Storms*, chapter iii), may contribute towards a fuller knowledge of stormy anticyclonism. It will also be interesting and instructive to see how readily Köppen's theorems can be deduced from Ferrel's laws.

405. *Solar-Barometric Virials.*

The first physical paper which I communicated to the American Philosophical Society (*Proc. A. P. S.*, ix., 283-8) was based on virial considerations, but the discussions of Clausius had not prepared the way for their general acceptance. Accordingly, the method of treatment was so new, that many persons looked upon the results merely as curious and, perhaps, accidental coincidences. The foregoing relations of virial influence to time of rotary oscillation enable us to reach the same results in a more simple way.

The mean barometric fluctuations, both daily and annual, may be regarded as functions of time, mass and distance. The mean daily disturbances take place at $r_3 = 3962.8$ miles from the virial centre; the mean annual disturbances at ρ_3 = Earth's semi-axis major from their virial centre. The disturbed atmospheric mass and the equilibrating value of g are the same in both cases. The virtual potential of daily rotation is $\frac{g}{2} \times 86164.08^2 =$ 22,559,593.75 miles. Gen. Sabine's means of five years' observations at St. Helena, show a daily barometric range of .067 in., and an annual range of .135 in. (see *Proc. Am. Phil. Soc.*, x, 375, foot-note). The geographical, magnetic and climatic situation of St. Helena is such as to give the following simple harmonic approximation for ρ_3 (Note 377; 5).

$$.067^2 : 135^2 : : 22,559,593.75 : 91,590,200 \text{ miles.}$$

406. *Encke's Comet.*

Dr. O. Backlund (*Copernicus*, Feb. 1883; cited in *Science*, **531**), says that "the investigations hitherto made of the theory of Encke's comet really prove nothing as to the existence of a resisting medium in space. Even if we should succeed by such an hypothesis to explain sufficiently the increase of the mean motion and the decrease of the eccentricity during the period 1819–48, a simple hypothesis like this will not at the same time suffice for the motion of the comet after 1865, as the variation of the mean movement after that time has most probably become different. Not until the period 1865–81, and its connection with the earlier one have been fully discussed, will it perhaps become possible to find indications of the nature of the unknown forces which act on the comet." If an æthereal medium is set in vibration by the passage of comets or other cosmical bodies, there will be, as in the case of tidal disturbances, both accelerating and retarding influences. We must know more than we now do, of the nature of the medium as well as of the laws of elasticity, before it will be safe to dogmatize about a resisting medium or about the second law of thermodynamics. The equality of action and reaction may, perhaps, set limits both to nucleal condensation and to æthereal expansion, the two limits being opposite phases of cyclical changes which all matter is always undergoing. The unity of energy which is indicated by æthereal relations of mass and velocity (Notes 388, 400), gives great likelihood to this hypothesis.

407. *Sound-Spectra.*

Frazer's "Examination of the phonograph record under the microscope" (*Jour. of the Franklin Inst.*, lxxv, 348; *Proc. Am. Phil. Soc.*, xiii, 531), showed that each of the alphabetic sounds has a special combination of vibrations, which may be visibly impressed upon a metallic sheet. The harmonic correspondence between the wave-lengths of musical notes and those of the principal lines in the visible spectrum (*Proc. Am. Phil. Soc.*, xiii, 149), increases the probability that there may be an unbroken series of waves, from the lowest audible sound to the highest actinic impulse. Mayer's experiments with the antennæ of mosquitos and Langley's observations of absorption bands (Note 401), approximate the gamuts of light and sound and suggest the desirableness of some more sensitive method for recording audible waves and interferences than is furnished by the phonograph. The radial virials of light and the tangential virials of sound (Note 390) furnish a field for research which is almost wholly unexplored. In view of the wonderful advance of spectral photography during the last decade, we may venture to hope that the record may sometime be extended so as to include the interferences of sound-waves.

408. *Investigators of Spectral Harmony.*

The earliest indications of harmony in spectral lines of which I have found any record, were given by Prof. Gustavus Hinrichs, in the American Journal of Science for 1864 (vol. xxxviii, p. 31, seq). In the Comptes Rendus of the French Academy, for 1869 and 1870, Lecoq de Boisbaudran published several harmonies of a character analogous to those of Hinrichs, his first paper being deemed of so much importance that the Academy allowed its insertion without abridgment, although it exceeded the statutory length. He referred to a communication of Mascart, on the same subject, in August, 1868, and also to a *pli cacheté* of his own which was deposited in the archives of the Academy in 1865. G. Johnstone Stoney (*Rept. Brit. Assoc.*, 1870 ; *Proc. Roy. Irish Acad.*, 1871 ; *P. Mag.*, 1871) and J. L. Soret (*Bib. Universelle*, Sept. 15, 1871, cited in *P. Mag.*, 1871, xlii, 464) seem to have been next on the list. My own investigations began in 1864, with the study of "oscillations moving with the velocity of light" (*Proc. Am Phil. Soc.*, ix., 408), but my first indications of harmonic wave-lengths were not published until 1873 (*Ib.*, xiii, 150). Guided by a conviction of the physical necessity that all æthereal undulations must be harmonic, I have been led into the discovery of a great variety of spectral and other coördinated harmonies.

409. *Velocity of Wave Propagation.*

As there has been some misapprehension with regard to my deduction of the relation between the mean velocity of oscillating æthereal particles and the velocity of wave propagation (Note 384), it may be well to explain the ground on which it rests. In considering the "uniform wave of oscillation," in a star which is rotating under the condition that $g_0 t_0 = v_\lambda$ (Note 379), the *vis viva* of a revolving particle at l (Note 381), is $\frac{1}{2}$ as great as the *vis viva* of the same particle from the indefinite fall* which has produced central condensation. *Vis viva* varies as distance of possible projection against uniform resistance ; therefore l and $\frac{1}{2} l$ may be taken, respectively, as the measures of the virials of indefinite and of virtual fall. Hence arises a tendency to the formation of an oscillatory node at $\frac{1}{2} l$, together with a tendency to the radial projection of the node, in the equatorial plane, by

* This is rigidly true only when the fall is infinite, but in falling from Neptune to Sun the deviation from exactness would be less than $\frac{1}{128}$ of one per cent.

the centrifugal force of rotation. The direct and reciprocal centres of linear oscillation, at $\frac{2}{3}\, l$ and $\frac{1}{3}\, l$, tend to throw the node at $\frac{1}{2}\, l$ from or toward the centre. The reciprocal centre, $\frac{1}{3}\, l$, is pivotal in respect to the direct centre, $\frac{2}{3}\, l$, thus producing a secondary centre of linear oscillation at $\frac{5}{9}\, l$. This indicates the relative *vis viva* of radial projection which corresponds to an oscillatory tangential *vis viva* of l. The corresponding relative velocity is $\sqrt{\frac{5}{3}}$.

410. *Propagation of Explosive Waves.*

Berthelot and Vieille (*Ann. de Chim. et de Phys.*, xxviii, 293) give the equation $\theta_1 = \theta_0 \sqrt{\frac{Q + q}{q}}$, in which Q is the amount of heat set free at the moment of chemical combination; q, 273 times the specific heat; θ_1, the velocity of explosive translation of gaseous molecules; θ_0, the velocity of mean translation after the explosive wave has ceased to exert any influence. They have verified the formula approximately, for a score of gaseous mixtures of very various compositions. They think that in the act of explosion a certain number of molecules are thrown forward with all the velocity corresponding to the maximum temperature developed by the chemical combination; this movement is transmitted from one inflamed edge to another, in a wave which is propagated with a velocity either identical, or comparable, to that of the molecules themselves.

Introduction to a Study of the North American Noctuidæ. By A. R. Grote, A. M.

(Read before the American Philosophical Society, June 16, 1883.)

In my "List of the Noctuidæ," 1874, the "Check Lists" of 1876 and 1882, my "Illustrated Essay" and a number of different papers, I have explained the characters of *Noctuidæ*, a family of moths of nocturnal habit and of very general distribution. These structural features, which are used in establishing genera and other divisions are briefly summarized as follows, taking the three divisions of the body in turn:

I. The *Head:* character and structure of the compound eyes, which are either full or ovate, small, large, or more or less constricted, and have their surface naked or studded with hair, and the orbits sometimes provided with longer hair, dependent over the eye and called *lashes;* the character and structure of the clypeus or front, between the eyes, which is swollen or flat and sometimes provided with a tubercle, or horns of various shapes and sizes, or a depression; the presence of ocelli; the shape and size of palpi and tongue; the vestiture of the different parts.